Flush-Mounted Electrostatic Probe in the Presence of Negative Ions

K. J. Touryan*
Sandia Laboratories, Albuquerque, N. Mex.

AND

P. M. Chung†
University of Illinois at Chicago Circle, Chicago, Ill.

The current voltage characteristics of continuum flush-mounted electrostatic probes are analyzed in the presence of negative ions. For equal electron and heavy particle temperatures, negative ions modify the ambipolar region but leave the sheath solution unchanged. The results show that negative ions suppress the electron saturation current considerably and increase the positive ion saturation current slightly. Decreasing the negative ion mobility increases the positive saturation current and further suppresses the negative saturation current. The effect of unequal electron and ion temperatures is discussed in some detail.

Nomenclature

$C_{+,-,e}$ = positive ion, negative ion, and electron mass fraction, respectively
$D_{+,-,e}$ = positive ion, negative ion, and electron diffusion coefficients, respectively
e = electron charge
E = electric field
f = $\int (u/U_o)d\eta$
F = friction work in energy equation, also potential drop inside the sheath, see Eq. (40)
G,H = functions defined in Eq. (28)
I_+,I_- = positive and negative ion slopes at sheath edge, Eq. (26)
j = current density
k = Boltzmann constant
K = function defined in Eq. (28)
L,M,N = functions defined in Eq. (21) (L—also characteristic length)
M = particle mass
n = number density
Q_{in},Q_{el} = inelastic and elastic energy exchange terms
Q = function defined in Eq. (15)
R = electric Reynolds number $R = \epsilon L U_o/D_{+o}$
t = stretched coordinate [see Eq. (28)]
T = temperature
u,v = velocity components, x and y direction, respectively
x,y = coordinates
X = function defined in Eq. (29)
α = λ_D/L, nondimensional Debye length
β,β_-,β' = ratios; $\beta = D_{+o}/\epsilon D_{eo}$, $\beta_- = D_{+o}/D_{-o}$, $\beta' = (1-\beta)/1+\beta$
γ,γ',Γ_o = defined in Eq. (25)
$\Gamma_{+,-,e}$ = positive ion, negative ion, and electron fluxes, respectively
∇ = gradient operator
η = boundary-layer coordinate
ϵ = temperature ratio T_o/T_{eo}
ϵ_o = permittivity in vacuum
Φ = electric potential
ψ = nondimensional electric potential
μ = conductivity
ρ = mass density

Subscripts

$-,+,e$ = negative ion, positive ion, and electron, respectively
B = body surface
o,∞ = boundary-layer edge
s = species

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* Research Manager. Member AIAA.
† Professor. Member AIAA.

Introduction

THE flush-mounted or surface electrostatic probe is an attractive instrument for measuring certain plasma properties, such as the electron number density, in re-entry vehicle plasmas. As the name implies, the flush probe consists of a conductor surrounded by an insulator whose exposed surface is flush with that of the body. When the dimensions of the probe L are small compared to a mean free path λ, it acts as a classical Langmuir probe. For re-entry boundary-layer measurements, however, it is not practical to have $L \ll \lambda$. Furthermore, under most re-entry conditions the sheath, where charge separation is significant, is collision dominated.

Lam,[1] Chung,[2,3] Denison,[4] and others, have studied the interaction between a solid body and a weakly ionized flowing plasma under the aforementioned conditions. In particular, Lam[1] has outlined a general continuum theory for a flush probe where $\lambda_D/L \ll R^{1/2}$, and where the weakly ionized plasma consists of positive ions and electrons.

Certain ablating surfaces, however, introduce negative ions in the plasma flow and change the current-voltage characteristics of the probe significantly (see Ref. 5). To study this effect, we will follow Lam's[1] formulation and include negative ions in the weakly ionized flowing plasma. The general formulation will be for a compressible plasma flow, but solutions will be obtained for an incompressible flat plate flow only, in order to illustrate the effect negative ions have on the current-voltage characteristics of the probe.

General Formulation

The compressible steady-state species conservation equations for singly ionized plasma (negative ions, positive ions, and electrons) can be written as

$$\rho'\bar{v}'\cdot\nabla'(n_s/\rho') + \nabla'\cdot\bar{\Gamma}_s = \dot{n}_s \tag{1}$$

where $s = +, -,$ and e for positive ions, negative ions, and electrons, respectively. The flux Γ_s is given by

$$\bar{\Gamma}_+ = -\rho' D_+'\nabla'(n_+/\rho') - (eD_+'/kT')n_+\nabla'\Phi \tag{2}$$

$$\bar{\Gamma}_- = -\rho' D_-'\nabla'(n_-/\rho') + (eD_-'/kT')n_+\nabla'\Phi \tag{3}$$

$$\bar{\Gamma}_e = -\rho' D_e'\left(\frac{T'}{T_e'}\right)\nabla'\left(\frac{T_e'}{T'}\frac{n_e}{\rho'}\right) + \frac{eD_e'}{kT_e'}n_e\nabla'\Phi \tag{4}$$

Poisson's equation can be written as

$$\nabla'^2\Phi = -e/\epsilon_o(n_+ - n_- - n_e) \quad (5)$$

and the electron energy equation is given by

$$n_e(\bar{v}' + \bar{V}_e')\cdot\nabla'(\tfrac{5}{2}kT_e' - e\Phi) = \nabla'\cdot(\mu_e\nabla T_e') + \bar{F} + Q_{\text{in}} + Q_{\text{el}} \quad (6)$$

where $\bar{V}_e'$ is the electron diffusion velocity, $\bar{F}$ is the friction work and $Q_{\text{in}},Q_{\text{el}}$ inelastic and elastic energy exchange terms between electrons and heavy particles. If we introduce the definition of mass fraction in Eq. (1) viz., $C_s = M_s n_s/\rho'$, the result is

$$\rho'\bar{v}'\cdot\nabla'(C_s/M_s) + \nabla'\cdot\bar{\Gamma}_s = \dot{n}_s \quad (1a)$$

or, from conservation of mass, $\nabla\cdot\rho'\bar{v}' = 0$ we have

$$\nabla'\cdot[\rho'v'(C_s/M_s) + \bar{\Gamma}_s] = \dot{n}_s \quad (1b)$$

The current density then is given by

$$\bar{J} = e\{\bar{\Gamma}_+ - \bar{\Gamma}_- - \bar{\Gamma}_e + \rho'v'[(C_+/M_+) - (C_-/M_-) - (C_e/M_e)]\} \quad (7)$$

Equations (1b) and (7) imply $\nabla\cdot\bar{J} = 0$, if $\dot{n}_+ = \dot{n}_- + \dot{n}_e$. Also, Eq. (1) is valid for weakly ionized gases, where the electric field effects are decoupled from the gas dynamics of the flowfield. In nondimensional form, Eqs. (1a) and (5) are represented as follows for $\dot{n}_s = 0$:

$$R\rho\bar{v}\cdot\nabla C_+ - \nabla\cdot\{\rho D_+[\epsilon\nabla C_+ - (C_+/T)\nabla\psi]\} = 0 \quad (8)$$

$$\beta_-R\rho\bar{v}\cdot\nabla C_- - \nabla\cdot\{\rho D_-[\epsilon\nabla C_- + (C_-/T)\nabla\psi]\} = 0 \quad (9)$$

$$\beta R\rho\bar{v}\cdot\nabla C_e - \nabla\cdot\left\{\rho D_e\left[\left(\frac{T}{T_e}\right)\nabla\left(\frac{T_e}{T}C_e\right) + \frac{C_e}{T_e}\nabla\psi\right]\right\} = 0 \quad (10)$$

$$\alpha^2\nabla^2\psi = \rho[C_+/M_+ - C_-/M_- - C_e/M_e] \quad (11)$$

where $R = \epsilon LU_o/D_{+o}$ is often called the electric Reynolds number R and for most re-entry plasmas is of order Re, the viscous Reynolds number; $\epsilon = T_o/T_{eo}$; $\psi = -e\Phi/kT_{eo}$; $\beta_- = D_{+o}/\epsilon D_{eo}$ and is of order unity, $\beta = D_{+o}/\epsilon D_{eo}$ and, in general, is much less than unity; $\alpha = \lambda_D/L$ and $\lambda_D = (\epsilon_o kT_{eo}M_e/e^2\rho_oC_{eo})^{1/2}$ is the Debye length. Subscript o represents a reference condition, e.g., boundary-layer edge.

If we let $T = T_e$ ($\epsilon = 1.0$) and follow Lam,[1] we can eliminate C_e from Eq. (10) and after some algebraic manipulations obtain the following set of equations:

$$R\rho\bar{v}\cdot\nabla[(1+\beta)C_+ + (\beta_- - \beta)C_-] - \nabla\cdot\{\rho(D_e + D_+)\nabla C_+ + \rho(D_- - D_e)\nabla C_- - \rho/T\nabla\psi[C_+(D_+ - D_-) - C_-(D_- - D_e)]\} = \alpha^2Q \quad (12a)$$

$$R\rho\bar{v}\cdot\nabla(\beta_- - \beta)C_- - \nabla\cdot\{\rho(D_e - \beta D_+)\nabla C_+ + \rho(D_- - D_e)\nabla C_- + \rho/T\nabla\psi[C_-(D_- - D_e) + C_+(D_e + \beta D_+)]\} = \alpha^2Q \quad (13a)$$

$$\beta_-R\rho\bar{v}\cdot\nabla C_- - \nabla\cdot[\rho D_-\nabla C_- + \rho(D_-C/T)\nabla\psi] = 0 \quad (14a)$$

where

$$\alpha^2Q = \alpha^2\left\{\nabla\cdot\left[\frac{D_e}{T}\nabla^2\psi\nabla\psi + \rho D_e\nabla\left(\frac{1}{\rho}\nabla^2\psi\right)\right] - \beta R\rho\bar{v}\cdot\nabla\left(\frac{1}{\rho}\nabla^2\psi\right)\right\} \quad (15)$$

(In all these equations and in subsequent ones $C_{+,-}/M_{+,-} = C'_{+,-}$ and the primes are deleted.) It is not hard to see that Eq. (13) represents the current density defined by Eq. (7). Also, Eqs. (12–15) are exact in the framework of the present theory. Now, if the variations with respect to temperature of the diffusion coefficients, D_+', D_-', and D_e', are similar, then $D_+ = D_+'/D_{+o}'$, $D_- = D_-'/D_{-o}'$, and $D_e = D_e'/D_{eo}'$ will all be about the same at each point of the flowfield. The differences of $(D_+ - D_-)$ and $(D_- - D_e)$ will then be negligible throughout the boundary layer, and the Eqs. (12–15) become considerably simplified, viz.,

$$R\rho\bar{v}\cdot\nabla[(1+\beta)C_+ + (\beta_- - \beta)C_-] - \nabla\cdot[\rho(D_e + D_+)\nabla C_+] = \alpha^2Q \quad (12b)$$

$$R\rho\bar{v}\cdot\nabla(\beta_- - \beta)C_- - \nabla\cdot\left[\rho(D_e - \beta D_+)\nabla C_+ + \frac{\rho}{T}C_+(D_e + \beta D_+)\nabla\psi\right] = \alpha^2Q \quad (13b)$$

$$\beta_-R\rho\bar{v}\cdot\nabla C_- - \nabla\cdot[\rho D_-\nabla C_- + \rho(D_-C_-/T)\nabla\psi] = 0 \quad (14b)$$

Ambipolar Region

For a thin sheath, $\alpha \to 0$ and charge separation is confined to a very thin region near the wall. The rest of the boundary layer can be simplified to represent the ambipolar region where quasi-neutrality holds. For a two-dimensional flow Eqs. (12b–14b) becomes for the ambipolar region

$$\rho u\frac{\partial}{\partial x}(C_+ + \gamma C_-) + \rho v\frac{\partial}{\partial y}(C_+ + \gamma C_-) = \frac{\partial}{\partial y}\left[\rho\frac{D_+ + D_e}{(1+\beta)R}\right]\frac{\partial C_+}{\partial y} \quad (16)$$

$$\rho u\frac{\partial}{\partial x}(\beta_- - \beta)C_- + \rho v\frac{\partial}{\partial y}(\beta_- - \beta)C_- = \frac{\partial}{\partial y}\left(\frac{D_e + \beta D_+}{R}\frac{\rho}{T}C_+\frac{\partial\psi}{\partial y} + \frac{D_e - \beta D_+}{R}\rho\frac{\partial C_+}{\partial y}\right) \quad (17)$$

$$\rho u\frac{\partial C_-}{\partial x} + \rho v\frac{\partial C_-}{\partial y} = \frac{\partial}{\partial y}\left(\frac{D_-}{\beta_-R}\frac{\rho}{T}C_-\frac{\partial\psi}{\partial y} + \frac{D_-}{\beta_-R}\rho\frac{\partial C_-}{\partial y}\right) \quad (18)$$

where $\gamma = (\beta_- - \beta)/1 + \beta$.

If we multiply Eq. (18) by $(\beta_- - \beta)$ and subtract from Eq. (17), we obtain

$$\frac{\partial}{\partial y}\left\{\left[\frac{\rho}{T}(D_e + \beta D_+)C_+ - \frac{\rho}{T}\frac{D_-}{\beta_-}(\beta_- - \beta)C_-\right]\frac{\partial\psi}{\partial y} + (D_e - \beta D_+)\rho\frac{\partial C_+}{\partial y} - \frac{\beta_- - \beta}{\beta_-}D_-\rho\frac{\partial C_-}{\partial y}\right\} = 0 \quad (19)$$

which can be integrated once to yield

$$\frac{\rho}{T}\left[(D_e + \beta D_+)C_+ - D_-\frac{\beta_- - \beta}{\beta_-}C_-\right]\frac{\partial\psi}{\partial y} + (D_e - \beta D_+)\rho\frac{\partial C_+}{\partial y} - \frac{\beta_- - \beta}{\beta_-}D_-\rho\frac{\partial C_-}{\partial y} = A(x) \quad (20)$$

where

$$A(x) = \left[(1+\beta)C_{+o} - \frac{\beta_- - \beta}{\beta_-}C_{-o}\right]\frac{\partial\psi}{\partial y}\bigg|_{y=y_o}$$

or

$$\frac{\rho}{T}\frac{\partial\psi}{\partial y} + \frac{\rho}{LC_+ - MC_-}\left(N\frac{\partial C_+}{\partial y} - M\frac{\partial C_-}{\partial y}\right) = \frac{A(x)}{LC_+ - MC_-} \quad (21)$$

where $L = D_e + \beta D_+$, $M = 1 - \beta/\beta_-D_-$, $N = D_e - \beta D_+$.

If now Eq. (21) is substituted into Eq. (17), two equations are obtained, explicit in C_+ and C_-, thus

$$\rho u\frac{\partial}{\partial x}(C_+ + \gamma C_-) - \rho v\frac{\partial}{\partial y}(C_+ + \gamma C_-) = \frac{\partial}{\partial y}\left[\rho\frac{D_e + D_+}{(1+\beta)R}\frac{\partial C_+}{\partial y}\right] \quad (22)$$

$$\rho u \frac{\partial}{\partial x}(\beta_- - \beta)C_- + \rho v \frac{\partial}{\partial y}(\beta_- - \beta)C_- = \frac{\partial}{\partial y}\left(\rho \frac{N}{R}\frac{\partial C_+}{\partial y}\right) + \frac{\partial}{\partial y}\left[\frac{LA(x)C_+}{R(LC_+ - MC_-)} - \frac{L\rho C_+}{R(LC_+ - MC_-)}\left(N\frac{\partial C_+}{\partial y} - M\frac{\partial C_-}{\partial y}\right)\right] = 0 \quad (23)$$

Note that the effect of the negative ions is not merely to alter the ambipolar diffusion coefficient. The boundary conditions are: $y \to \infty$; $u \to U_o, C_+ \to C_{+o}, C_- \to C_{-o}$. $y \to 0$; $C_+ = C_- = 0$.

From Eq. (21) it is clear that $\partial\psi/\partial y \to \infty$ as $y \to 0$. The reason for this singular behavior near $y = 0$ is that in the vicinity of the wall $\alpha^2 Q$ is no longer small, and charge separation is significant. For a uniformly valid solution all the way to the wall, one has to construct the sheath solution and *match* it to the ambipolar solution at some $y = y^o$. However, if the sheath is very thin, y^o can be set equal to zero for calculating C_+ and C_-.

As an example, we will solve Eqs. (22) and (23) for the incompressible case, corresponding to the flow of a weakly ionized gas over a flat plate. After introducing the boundary-layer coordinate $\eta = y(U_o/D_+x)^{1/2}$ and setting the ion Schmidt number ν/D_+ equal to one, Eqs. (22) and (23) become

$$f d/d\eta(C_+ + \gamma C_-) + 4/(1+\beta)d^2C_+/d\eta^2 = 0 \quad (24)$$

$$f\frac{d}{d\eta}(\gamma C_-) + 2\beta'\frac{d^2C_+}{d\eta^2} + 2\frac{d}{d\eta}\left[\frac{C_+\Gamma_o}{C_+ - \gamma' C_-} - \frac{C_+}{C_+ - \gamma' C_-}\left(\beta'\frac{dC_+}{d\eta} - \gamma'\frac{dC_-}{d\eta}\right)\right] = 0 \quad (25)$$

where

$$f' = U/U_o \quad \gamma = \frac{\beta_- - \beta}{1+\beta} \quad \gamma' = \frac{\gamma}{\beta_-} \quad \Gamma_o = [1 - \gamma' C_o]\left.\frac{d\psi}{d\eta}\right|_o, \quad \beta' = \frac{1-\beta}{1+\beta}$$

It is clear from Eqs. (21), (24), and (25)‡ that in the vicinity of the wall, $\eta \to \eta^o(x)$, convective effects can be neglected, and we obtain immediately

$$C_+ \simeq I_+(\eta - \eta^o) \qquad C_- \simeq I_-(\eta - \eta^o) \quad (26)$$

$$\frac{\partial\psi}{\partial\eta} \simeq \frac{1}{(\eta - \eta^o)}\left(\frac{1 - \gamma' C_{-o}}{I_+ - \gamma' I_-}\left.\frac{\partial\psi}{\partial\eta}\right|_o - \frac{\beta' I_+ - \gamma' I_-}{I_+ - \gamma' I_-}\right) \quad (27)$$

Equations (26) and (27) will serve as boundary conditions for the sheath region.

Sheath Region

In the spirit of the aforementioned simplification, we now look at the incompressible sheath in the presence of negative ions. Once more, following Lam[1] we introduce the transformation

$$\eta = \eta^o(x) + (R\alpha^2/I_+)^{1/3}t \quad (28a)$$

$$C_+ = (R\alpha^2 I_+^2)^{1/3}K(t) \quad (28b)$$

$$C_- = (R\alpha^2 I_-^2)^{1/3}H(t) \quad (28c)$$

$$C_e = (R\alpha^2 I_e^2)^{1/3}G(t) \quad (28d)$$

If we substitute Eq. (28) into Eqs. (14) and (15), neglect convection terms and follow Lam, we obtain the following sheath equation:

$$d^2E/dt^2 = (E^3/2) + 2Et + 2X \quad (29)$$

where

$$X = (\beta' I_+ - \gamma' I_-)/(I_+ - \gamma' I_-) - (1 - \gamma' C_{-o})/(I_+ - \gamma' I_-)\,\partial\psi/\partial\eta|_o$$

$$E = -\partial\psi/\partial t$$

This equation is identical to Lam[1] with $\epsilon = 1$ (see Eq. 6.2, Ref. 2). It was studied in detail by Cohen.[6] The only difference lies in the value of the parameter X which now depends on the positive ion slope I_+ as well as the negative ion slope I_-. For $\epsilon \neq 1$ but T_e/T = constant throughout, the sheath equation changes and is now represented by the set

$$(1+\epsilon)K - (1-\epsilon)(I_-/I_+)^{2/3}H = (dE/dt) + (E^2/2) + (1+\epsilon)t - (1-\epsilon)t(I_-/I_+) \quad (30a)$$

$$\epsilon(dH/dt) + HE = (I_-/I_+)^{1/3}(\epsilon + X') \quad (30b)$$

$$\epsilon(dK/dt) - KE = \epsilon - X' \quad (30c)$$

where

$$X' = \frac{(1 - \gamma' C_{-o})(n\cdot\nabla\psi_o)}{R^{1/2}(I_+ - \gamma' I_-)} - \frac{1}{(1+\beta)(I_+ - \gamma' I_-)} \times \left[(1 - \epsilon\beta)I_+ - \frac{(\beta_- - \epsilon\beta)}{\beta_-}I_-\right]$$

After considerable manipulations, one can eliminate k and H from Eq. (30) and obtain the following sheath equation:

$$\epsilon(E''/E)' + \epsilon E'' - (E^2/2\epsilon)' - A(E'/E^2) - (E^3/2\epsilon) - BtE - C = 0 \quad (31)$$

where A, B, and C are functions of ϵ, I_-/I_+ and X'.

The boundary conditions for Eq. (29) are as follows: on the body surface, we require that

$$K = G = H = 0 \quad (32a)$$

$$E' = \partial E/\partial t = 0 \quad (32b)$$

As $t \to \infty$, we require that the sheath solutions match with the ambipolar solutions given in Eq. (27):

$$E(t \to \infty) = X/t \quad (33)$$

After $E(t,X)$ has been obtained, the distribution of ψ in the sheath is given by

$$\psi = \psi_w(x) - \int_{t_w}^{t} E(t,X)dt \quad (34)$$

Inasmuch as the previous analysis is identical to Lam,[1] we refer the reader to Ref. 1 for further details. For the compressible sheath where the temperature T is not a constant, but is equal to T_e, one still obtains Eq. (29). However, the potential ψ is now obtained from the expression

$$\psi = \int TEdt$$

and the coordinate t and the parameter X are related to the general compressible flow transformation coordinates η and ξ.[4]

Current Voltage Characteristics

The current voltage characteristics for the incompressible flow are now given by

$$\psi = \underbrace{\int_0^{\eta^o}\frac{d\psi}{dt}dt}_{\text{sheath}} + \underbrace{\int_{\eta^o}^{\eta}\frac{d\psi}{d\eta}d\eta}_{\text{ambipolar}} + \underbrace{\int d\bar{r}\cdot\nabla\psi}_{\text{outer}} \quad (35)$$

‡ In Eqs. (24) and (25) $\beta \ll 1$, e.g., for NO^+, $\beta = 0.0043$. On the other hand β_- can be as high as 2 for SF_6^- and as low as 0.5 for OH^-. Under these conditions, $\gamma' \simeq \beta' \approx 1$.

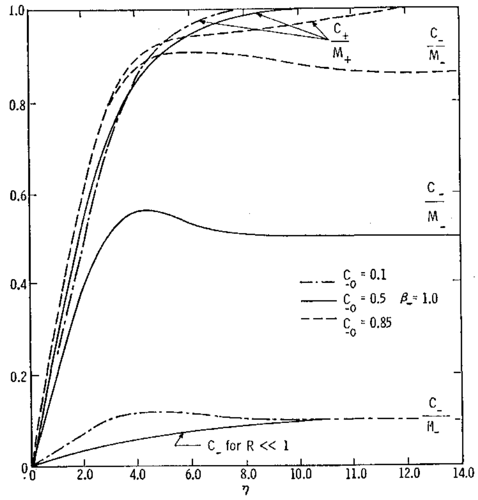

Fig. 1 The effect of changing negative ion concentration on the ambipolar region.

Following Lam[1] we have,

$$\psi_{\text{sheath}} = \psi_{\text{wall}} - X\ \ln t - F(X);\ F(X) = \int_{t_{\text{wall}}}^{1} E dt + \int_{1}^{\infty}\left(E - \frac{X}{t}\right) dt \quad (36)$$

$$\psi_{\text{amb}} \simeq \psi_o(X) - X\ \ln t - \tfrac{1}{3}\ ln R\alpha^2(I_+^2 - I_-^2) + \frac{1 - \gamma' C_o}{I_+ - \gamma' I_-}\frac{\partial\psi}{\partial\eta}\bigg|_o\left(\int \frac{\beta' I_+ - \gamma' I_-}{I_+ - \gamma' I_-} d\eta - ln t\right) \quad (37)$$

For the outer solution we get

$$J_B = e[\Gamma_e + \Gamma_- - \Gamma_+] = -\frac{eD_e n_o}{L}[(C_e + \beta_- C_- + \beta C_+)\nabla\psi + \nabla(\beta C_+ - \beta_- C_- - C_e)] \quad (38)$$

At the edge of the ambipolar region $C_e + C_- = C_+ = 1$ and $C_- = C_{-o}$, then

$$J_B \simeq -J_\infty\left(1 - \frac{1 - \beta_-}{1 + \beta} C_\bullet\right)\frac{\partial\psi}{\partial y}\bigg|_o \simeq = J_\infty(1 - \gamma' C_{-o})\frac{\partial\psi}{\partial y}\bigg|_o \quad (39)$$

where $J_\infty = (1 + \beta)eD_e n_o/L$.

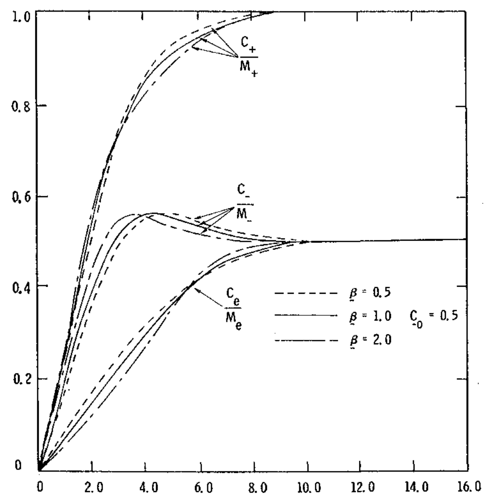

Fig. 2 The effect of changing β_- ($=D_+/D_-$) for a given negative ion mass fraction C_o at boundary-layer edge.

Substituting Eqs. (36–39) in Eq. (35) we finally get

$$\psi_{\text{wall}} \simeq \frac{1}{j_3}\frac{J_B}{J_\infty} + \left\{\frac{1}{3}\left[\frac{\beta' I_+ - \gamma' I_-}{I_+ - \gamma' I_-} + \frac{1}{I_+ - \gamma' I_-}\frac{J_B}{J_\infty(R)^{1/2}}\right]\ln\frac{1}{R\alpha^2(I_+^2 - I_-^2)} - \frac{1}{I_+ - \gamma' I_-}\frac{J_B}{(R)^{1/2}J_\infty}(1 + D)\right\} + F\left[\frac{\beta' I_+ - \gamma' I_-}{I_+ - \gamma' I_-} + \frac{1}{I_+ - \gamma' I_-}\frac{J_B}{(R)^{1/2}J_\infty}\right] \quad (40)$$

where the function F describes the potential drop inside the sheath.

The floating potential is obtained by setting $J_B = 0$. In dimensional form this gives

$$-\Phi_B = \frac{kT_e}{e}\left[\frac{1}{3}\frac{\beta' I_+ - \gamma' I_-}{I_+ - \gamma' I_-}\ln\frac{1}{R\alpha^2(I_+^2 - I_-^2)} + F\left(\frac{\beta' I_+ - \gamma' I_-}{I_+ - \gamma' I_-}\right)\right] \quad (40a)$$

In Eq. (40a) $\gamma' \simeq \beta' \simeq 1$ and, therefore, a plot of Φ_B vs $lnR\alpha^2(I_+^2 - I_-^2)$ will yield a slope equal to $\frac{1}{3}kT_e/e$, as in the case where negative ions are absent. Since $F \to \pm\infty$ as $X \to \pm 1$, the saturation currents are given by

$$J_B(\text{positive}) = J_\infty R^{1/2} I_+ 2\beta/(1 + \beta) \quad (41)$$

$$J_B(\text{negative}) = J_\infty R^{1/2} 2/(1 + \beta)\{[(\beta_- - \beta)/\beta_-]I_- - I_+\} \quad (42)$$

For $C_- = 0$, Eqs. (40–42) reduce to those of Lam.[1] In the limit of $C^o = 0$, $J_B(+)/J_B(-) = \beta_-$ as expected. For the thin sheath case, the values of I_+ and I_- can be obtained from a numerical solution of Eqs. (24) and (25), with γ and Γ_o as parameters. The solution was obtained for a flat plate, as an initial value problem using a shooting method. Table 1 summarizes the results for various values of C_o and β_-. Values used for the parameter Γ_o varied between $-0.02 \leq \Gamma_o \leq 0.45$. In the limiting cases of zero or hundred percent negative ions, the solution of course is independent of Γ_o. In Table 1 and Figs. 1 and 2, $\Gamma_o = 0$.

Discussion

Figures 1 and 2 show C_+ and C_- concentration profiles for various values of C_o and β_-, respectively. These curves show clearly that as the negative ion concentration is increased, the ambipolar diffusion layer becomes thicker and the positive ion slope near the wall increases. Figure 3 is a sketch of the current voltage characteristics of the flush probe as a function of negative ion concentration.

Table 1 Ion slopes and ratio of saturation currents for $\beta = \frac{1}{235}$ and $\epsilon = 1.0$

β_-	C_{-o}	I_+	I_-	$J_B(-)/J_B(+)$
0.5	0	0.257	0	235
	0.1	0.261	0.042	197
	0.5	0.272	0.188	72.7
	0.7	0.279	0.246	27.8
	1.0	0.30	0.30	0.50
1.0	0	0.257	0	235
	0.1	0.266	0.048	192
	0.5	0.280	0.211	57.9
	0.7	0.294	0.272	17.6
	1.0	0.315	0.315	1.0
2.0	0	0.257	0	235
	0.1	0.264	0.056	185
	0.5	0.293	0.240	42.5
	0.7	0.317	0.305	8.9
	1.0	0.335	0.335	2.0

It is evident from these results that the presence of negative ions suppresses the electron saturation current, and increases the positive ion saturation current slightly. The effect of increasing β_- (decreasing the negative ion mobility) is to increase the positive saturation currents and decrease the negative saturation current. These results are in good qualitative agreement with the measurements of Starner[5] who used a flowing argon plasma with various quench gases, such as SF_6, CF_4, etc., added to the plasma stream.

Figures 1 and 2 show the rather unexpected peaks in the negative ion concentrations within the boundary layer. These peaks are due to the following.

Equation (21) shows that the electric field intensity in the ambipolar region is proportional to $[A(x) - (\partial C_e/\partial y)]/C_e$ as it is when there are no negative ions (see Refs. 1 and 2). This quantity is equal to $[A(x) - (\partial C_+/\partial y)]/C_+$ when the negative ion concentration is zero. Hence, the electric field is negative throughout the boundary layer when $A = 0$, that is when the surface is electrically floating. The intensity of the negative electric field increases toward the surface as $\partial C_e/\partial y/C_e$ becomes larger.

This negative electric field, in addition to retarding the electron flux to the wall, would reduce the electron concentration near the wall by preventing much of the light electrons from reaching there.

When the negative-ion concentration is zero, however, the plasma (quasi) neutrality condition of the ambipolar region requires that $C_e = C_+$. Since the body force on the positive ions is in the direction opposite to that on the electrons, the electric field intensity cannot substantially reduce the electron concentration near the wall.

When there exist negative ions, on the other hand, the negative electric field successfully reduces the electron population in the region near the wall, say for $\eta < 4$ or 5, by increasing the negative-ion concentration to preserve the plasma neutrality. The negative-ion concentration increases in this region as the convection piles up the negative ions against the strong repulsive electric field. Without the convection, therefore, the peaks in the negative-ion concentrations shown in Figs. 1 and 2, and the corresponding reductions in the electron concentrations, will not be observed.

When the surface is negatively biased with respect to the inviscid stream, that is when $A < 0$, the electric field in the boundary layer will be more strongly negative than when $A = 0$. For such cases, therefore, the solution of Eqs. (24) and (25) should show more pronounced peaks in the negative-ion concentrations than those shown in Figs. 1 and 2. Similarly, when surface is positively biased ($A > 0$), the negative intensity of the electric field will be reduced. The peaks in the negative-ion concentration will then be correspondingly reduced and will eventually disappear as A is continuously increased.§

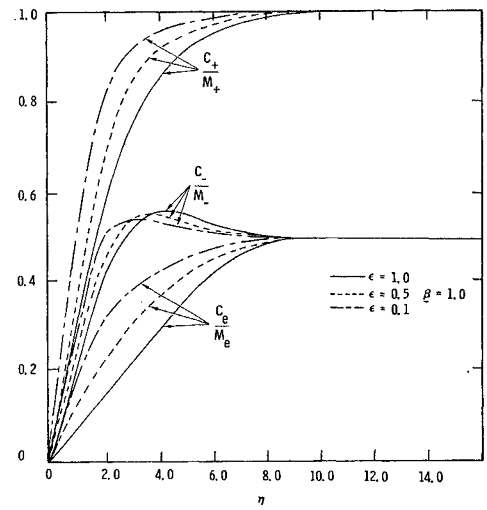

Fig. 4 The effect of unequal electron and ion temperatures, $\epsilon \neq 1$, on the ambipolar region.

It is noted here that the governing Eqs. (24) and (25) for the ambipolar region are functions of the parameter A when there exist negative ions. When the negative ions are absent, these equations degenerate to the single ambipolar equation for $C_+(= C_e)$ derived in Refs. 1 and 2, which is independent of A.

Figure 2 shows that the peak of negative ions exists further away from the wall as β_- is decreased. Decreasing of β_- implies an increase in D_-, and hence, an increase in K_-. Therefore, the opposition to the negative ion transport becomes more efficient as β_- is decreased. Consequently, with a decreased β_-, the piling up of the ions take place at a larger η where the electric field is weaker and at the same time the convective transport is stronger.

In the ambipolar region, the governing equations, Eqs. (12b–14b) and Eq. (11) for $\alpha^2 = 0$, can be reduced to the following set of equations if and only if the Reynolds number is zero.

$$\nabla^2(C_+/C_{+o}) = 0 \tag{43}$$

$$\nabla^2(C_-/C_{-o}) = 0 \tag{44}$$

$$\nabla^2(C_e/C_{eo}) = 0 \tag{45}$$

In the aforementioned equations, the gas density and the other properties are assumed to be constant. The boundary conditions for Eqs. (43–45) corresponding to those following Eqs. (22) and (23) are that the values C_+/C_{+o}, C_-/C_{-o}, etc., should be one at infinity, whereas they should be zero at the probe surface.

Equations (43) and (45) and their boundary conditions are identical to those for the plasmas with no negative ions when the Reynolds number is zero. (See, for instance, Ref. 6). Therefore, if there is no convection, the positive ion and electron concentration profiles, respectively normalized to their values at infinity, are unaltered by the presence of the negative ions. The solution of the problem is then trivial.

The convection, however, causes a rather fundamental change in the structure of the ambipolar region when there are negative ions, as was shown by the present solutions for high Reynolds numbers.

The effect of varying the ion to electron temperature ratio ϵ on the structure of the ambipolar region is shown on Fig. 4.

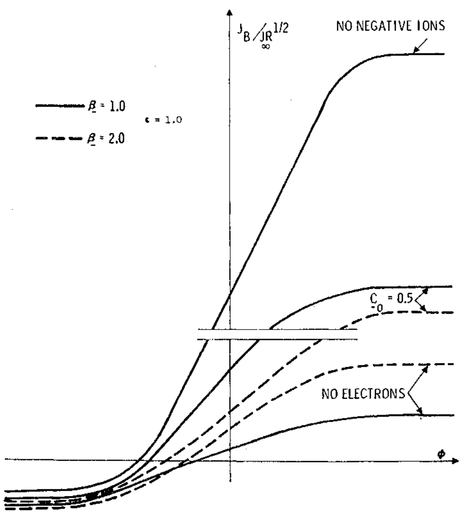

Fig. 3 Current-voltage characteristics (arbitrary units) for various values of β_- and C_o.

§ For $\Gamma_o = -0.01$ and $C_{-o} = 0.5$, the peak value for C_- increased by 2% over the $\Gamma_o = 0$ value. For Γ_o (or A) $= 0.01$ the reverse was true.

The decreasing ϵ can be thought of as the decreasing ion temperature instead of the increasing electron temperature. A decrease in the ion temperature, in the present formulation, implies a decrease in D_- and K_-. A reduced K_- causes the negative ion peak to move closer to the wall as was explained earlier in connection with the effect of varying β_-. This variation of the position of the negative ion peak with respect to ϵ is seen in Fig. 4.

Conclusions

On the basis of the previous simplified analysis, it is obvious that when negative ions are present in the boundary layer in quantities greater than trace amounts and $T_e \neq T_i$, one cannot determine the electron density concentration or the electron temperature from electrostatic probe measurements alone. For $T_e = T_i$, the *complete* current-voltage characteristics must be obtained before n_+ and T_e can be determined from Eqs. (40a), (41), and (42).

Appendix

For the case where the flow is incompressible but $\epsilon \neq 1$ (i.e., $T_e \neq T_i$), Eqs. (24) and (25) become

$$f\frac{d}{d\eta}(C_+ + \gamma C_-) + \frac{2(1+\epsilon)}{1+\beta}\frac{d^2C_+}{d\eta^2} - \frac{2(1-\epsilon)}{1+\beta}\frac{d^2C_-}{d\eta^2} = 0 \quad \text{(A1)}$$

$$f\frac{d}{d\eta}(\gamma C_-) + \frac{2(1-\beta\epsilon)}{1+\beta}\frac{d^2C_+}{d\eta^2} = -2\frac{d}{d\eta}\left\{\frac{C_+\Gamma_o}{C_+ - \gamma' C_-} - \frac{C_+}{C_+ - \gamma' C_-}\left[\frac{(1-\epsilon\beta)}{1+\beta}\frac{dC_+}{d\eta} - \frac{(\beta_- - \epsilon\beta)}{\beta_-(1+\beta)}\frac{\partial C_-}{\partial\eta}\right]\right\} = 0 \quad \text{(A2)}$$

The corresponding expression for $\partial\psi/\partial\eta$ near the wall yields

$$\frac{\partial\psi}{\partial\eta} \simeq \frac{1}{\eta - \eta^*}\left[\frac{1-\gamma' C_o}{I_+ - \gamma' I_-} - \frac{(1-\epsilon\beta)I_+}{(1+\beta)(I_+ - \gamma' I_-)} + \frac{(\beta_- - \epsilon\beta)I_-}{\beta_-(1+\beta)(I_+ - \gamma' I_-)}\right] \quad \text{(A3)}$$

To compute saturation currents for $\epsilon \neq 1.0$, the limiting values of the function $F(\epsilon,X')$ must be known. Hence,

$$F(\epsilon,X') = \int_{t_w}^{1} E(t;\epsilon,X')\,dt + \int_1^\infty \left(E - \frac{X'}{t}\right)dt$$

[see Eqs. (33) and (34)]. This in turn would require the solution of the modified sheath equation, Eq. (31) [or Eqs. (30)]. If, for example, one lets $X' \to {}^{-}_{+\epsilon}R$¶ for $F \to \pm\infty$, then one has

$$J_B(\text{positive}) = J_\infty R^{1/2}\frac{1}{1+\beta}\left[(1+\beta)RI_+ - (1+\epsilon\beta)I_+ - \frac{(\beta_- - \beta)}{\beta_-}RI_- - \frac{(\beta_- - \epsilon\beta)}{\beta_-}I_-\right]$$

$$J_B(\text{negative}) = J_\infty R^{1/2}\frac{1+\epsilon}{1+\beta}\left[I_-\frac{B_-(1+\epsilon) - 2\epsilon\beta}{\beta_-(1+\epsilon)} - I_+\right]$$

These reduce to the results of Lam[1] for $I_- = 0$ (no negative ions).

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¶ $R = [\epsilon(I_-/I_+)(\epsilon - 1) + 1]$.